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# ON VON NEUMANN'S PARAMETER OF EXTREMAL SHRÖDINGER OPERATOR

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In this paper we study connections between extremal accretive quasi-self-adjoint dissipative extensions of a non-negative symmetric Shrödinger operator with deficiency indices $(1,1)$ on $L_2[\ell,+\infty)$ and the moduli of their von Neumann's parameters. It is shown that the modulus of the corresponding von Neumann's parameter belongs to the interval $[\kappa_0,1)$, where for $\kappa_0\ge 0$ is obtained a new formula in terms of the values of the Weyl-Titchmarsh function $m_\infty(-0)$ and $m_\infty(i)$. An example that illustrates the obtained results is presented.

***Keywords***: Shrödinger operator, von Neumann's parameter, sectorial operators, extremal operator, Weyl-Titchmarsh function.

**Introduction.** In the current paper we set focus on non-self-adjoint Shrödinger operators on $L_2[\ell,+\infty)$ of the form

$$\left\{\begin{array}{l} T_h y=-y''+q(x)y \\ hy(\ell)=y'(\ell) \end{array}\right. \tag{9.1}$$

where $\operatorname{Im} h>0$. We study a connection between the modulus of the von Neumann parameter of the extremal accretive operator $T_h$ and its boundary value $h$. It is established that the modulus of the corresponding von Neumann's parameter belongs to the interval $[\kappa_0,1)$, where for $\kappa_0$ is obtained an exact new formula in terms of the values of the Weyl-Titchmarsh function $m_\infty(-0)$, $m_\infty(i)$.

*This paper is dedicated to the centenary of Georgiy Dmitrievich Suvorov, a Correspondent Member of the Ukrainian Academy of Sciences, a remarkable Human Being, mathematician, philosopher, educator, and enlightener. Despite of many singularities of life at that time, he was not among those who, using words of the famous poet Владимир Маяковский, "...видя безобразие обеими глазами, пишет о прелестях лирических утех". His apostolic service, support and help for students, post graduate students and colleagues especially to those who are in a big trouble was enormous. Once the great poet Александр Пушкин wrote: "И долго буду тем любезен я народу, что чувства добрые я лирой пробуждал, что в мой жестокий век восславил я свободу и милость к падшим призывал". These words completely relate to a wonderful life of G.D. and his many good deeds that will not be forgotten.*

**Preliminaries.** Let $\dot A$ be a closed, densely defined, symmetric operator in a Hilbert space $\mathcal H$ with inner product $(f,g)$, $f,g\in\mathcal H$. Suppose also that $\dot A$ has the deficiency indices $(1,1)$. Any non-symmetric operator $T$ in $\mathcal H$ such that

$$\dot A\subset T\subset \dot A^*$$

is called (see [1]) a *quasi-self-adjoint extension* of $\dot A$.

Assume now that $T\ne T^*$ is a maximal dissipative extension of $\dot A$,

$$\operatorname{Im}(Tf,f)\ge 0,\quad f\in \operatorname{Dom}(T).$$

Since $\dot A$ is symmetric with the deficiency indices $(1,1)$, its dissipative extension $T$ is automatically quasi-self-adjoint [3], that is, $\dot A\subset T\subset\dot A^*$, and hence, (see [8])

$$g_+-\kappa g_-\in\operatorname{Dom}(T)\quad\text{for some } |\kappa|<1, \tag{9.2}$$

where $g_{\pm} \in \mathrm{Ker}\,(\dot{A}^* \mp iI)$ are normalized deficiency vectors of $\dot{A}$. Throughout this paper $\kappa$ will be referred to as the *von Neumann parameter* of operator $T$.

Recall that a quasi-self-adjoint extension extension $T$ of $\dot{A}$ in a Hilbert space $\mathcal{H}$ is called *accretive* [17] if $\mathrm{Re}\,(Tf, f) \geq 0$ for all $f \in \mathrm{Dom}(T)$. An operator $T$ is called *$\alpha$-sectorial* [16,17] if there exists a value of $\alpha \in (0, \pi/2)$ such that

$$(\cot\alpha)|\mathrm{Im}\,(Tf, f)| \leq \mathrm{Re}\,(Tf, f), \qquad f \in \mathrm{Dom}(T). \tag{9.3}$$

An accretive operator $T$ is called *extremal* if it is not $\alpha$-sectorial for any $\alpha \in (0, \pi/2)$.

**Quasi-self-adjoint extension of symmetric Schrödinger operator.** Let $\mathcal{H} = L_2[\ell, +\infty)$ and $l(y) = -y'' + q(x)y$, where $q(x)$ is a real locally summable function. Suppose that the symmetric operator

$$\begin{cases} \dot{A}y = -y'' + q(x)y \\ y(\ell) = y'(\ell) = 0 \end{cases} \tag{9.4}$$

has deficiency indices (1,1) and is defined on the set $D^*$ of functions locally absolutely continuous together with their first derivatives such that $l(y) \in L_2[\ell, +\infty)$ and the boundary conditions as in (9.4). Consider quasi-self-adjoint extensions of $\dot{A}$ defined on functions $y(x) \in D^*$ with the corresponding boundary conditions

$$\begin{cases} T_h y = l(y) = -y'' + q(x)y \\ hy(\ell) = y'(\ell) \end{cases}, \quad \begin{cases} T_h^* y = l(y) = -y'' + q(x)y \\ \overline{h}y(\ell) = y'(\ell) \end{cases}. \tag{9.5}$$

Let $\dot{A}$ be a symmetric operator of the form (9.4) , generated by the differential operation $l(y) = -y'' + q(x)y$. Let also $\varphi_k(x, \lambda)$, $(k = 1, 2)$ be the solutions of the following Cauchy problems:

$$\begin{cases} l(\varphi_1) = \lambda\varphi_1 \\ \varphi_1(\ell, \lambda) = 0 \\ \varphi_1'(\ell, \lambda) = 1 \end{cases}, \qquad \begin{cases} l(\varphi_2) = \lambda\varphi_2 \\ \varphi_2(\ell, \lambda) = -1 \\ \varphi_2'(\ell, \lambda) = 0 \end{cases}.$$

It is well known [18] that there exists a function $m_\infty(\lambda)$ for which

$$\varphi(x, \lambda) = \varphi_2(x, \lambda) + m_\infty(\lambda)\varphi_1(x, \lambda)$$

belongs to $L_2[\ell, +\infty)$. This function $m_\infty(\lambda)$ is called the *Weyl-Titchmarsh function*. In [3,5] the following function is associated to a dissipative operator $T_h$ ($\mathrm{Im}\,h > 0$)

$$W_\Theta(z) = \frac{\mu - h}{\mu - \overline{h}} \cdot \frac{m_\infty(z) + \overline{h}}{m_\infty(z) + h}, \tag{9.6}$$

where $\mu \in \mathbb{R} \cup \{\infty\}$. Also, it was established in [8] that if $W_\Theta(z)$ of the form (9.6) is associated with $T_h$ and $\kappa$ is the von Neumann parameter of $T_h$, then

$$|\kappa| = \frac{1}{|W_{\Theta_{\mu,h}}(i)|} = \left|\frac{\mu - \overline{h}}{\mu - h} \cdot \frac{m_\infty(i) + h}{m_\infty(i) + \overline{h}}\right| = \left|\frac{m_\infty(i) + h}{m_\infty(i) + \overline{h}}\right|. \tag{9.7}$$

It is known [18] that $-m_\infty(z)$ is a Herglotz-Nevanlinna function. Hence, taking into account that $\mathrm{Im}\,h > 0$ and $\mathrm{Im}\,m_\infty(i) < 0$ one can easily directly verify that indeed $|\kappa| < 1$ in (9.7).

**The modulus of von Neumann's parameter in extremal case.** Suppose that the symmetric operator $\dot{A}$ of the form (9.4) with deficiency indices (1,1) is non-negative, i.e., $(\dot{A}f, f) \geq 0$ for all $f \in \mathrm{Dom}(\dot{A})$. For the remainder of this paper we assume that $m_\infty(-0) < \infty$. Then according to [21,22] we have the existence of the operator $T_h$, ($\mathrm{Im}\, h > 0$) that is accretive and/or sectorial.

Now suppose $T_h$ is a Schrödinger operator defined by (9.5). The following theorems are the main results of this paper.

**Theorem 9.1.** *Let $\dot{A}$ be a nonnegative symmetric Schrödinger operator of the form (9.4) with deficiency indices $(1,1)$ in $\mathcal{H} = L^2[\ell,\infty)$. If a dissipative Schrödinger operator $T_h$, ($\dot{A} \subset T_h \subset \dot{A}^*$), of the form (9.5) with the modulus $\kappa$ of its von Neumann's parameter is extremal, then $\kappa_0 \le \kappa < 1$, where $\kappa_0 \ge 0$ is given by*

$$\kappa_0 = \sqrt{\frac{\sqrt{|m_\infty(i)|^2 - 2m_\infty(-0)\mathrm{Re}\, m_\infty(i) + m_\infty^2(-0)} + \mathrm{Im}\, m_\infty(i)}{\sqrt{|m_\infty(i)|^2 - 2m_\infty(-0)\mathrm{Re}\, m_\infty(i) + m_\infty^2(-0)} - \mathrm{Im}\, m_\infty(i)}}. \tag{9.8}$$

Proof. To simplify further calculations we set

$$m_\infty(i) = a - ib,\ b > 0,\ m_\infty(-0) = m,\ c = (a-m)^2,\ d = c + b^2 > 0. \tag{9.9}$$

Then (9.7) yields

$$\kappa = \left|\frac{m_\infty(i) + h}{m_\infty(i) + \overline{h}}\right| = \left|\frac{a - ib + \mathrm{Re}\, h + i\mathrm{Im}\, h}{a - ib + \mathrm{Re}\, h - i\mathrm{Im}\, h}\right| = \left|\frac{(a + \mathrm{Re}\, h) + i(\mathrm{Im}\, h - b)}{(a + \mathrm{Re}\, h) - i(\mathrm{Im}\, h + b)}\right|$$
$$= \sqrt{\frac{(a + \mathrm{Re}\, h)^2 + (\mathrm{Im}\, h - b)^2}{(a + \mathrm{Re}\, h)^2 + (\mathrm{Im}\, h + b)^2}},$$

or

$$\kappa^2 = \frac{(a + \mathrm{Re}\, h)^2 + (\mathrm{Im}\, h - b)^2}{(a + \mathrm{Re}\, h)^2 + (\mathrm{Im}\, h + b)^2}. \tag{9.10}$$

Suppose that $T_h$ is an extremal operator. Therefore (see [4, 19, 20]) $\mathrm{Re}\, h = -m_\infty(-0) = -m$. Applying this to (9.10) and using (9.9) notations gives

$$\kappa^2 = \frac{(a-m)^2 + (\mathrm{Im}\, h - b)^2}{(a-m)^2 + (\mathrm{Im}\, h + b)^2} = \frac{c + (\mathrm{Im}\, h)^2 - 2b\,\mathrm{Im}\, h + b^2}{c + (\mathrm{Im}\, h)^2 + 2b\,\mathrm{Im}\, h + b^2}$$
$$= \frac{(\mathrm{Im}\, h)^2 - 2b\,\mathrm{Im}\, h + d}{(\mathrm{Im}\, h)^2 + 2b\,\mathrm{Im}\, h + d}.$$

Let us consider $\kappa^2$ as a function $f$ of $x = \mathrm{Im}\, h$, that is

$$f(x) = \kappa^2(x) = \frac{x^2 - 2bx + d}{x^2 + 2bx + d}. \tag{9.11}$$

Taking the derivative of $f(x)$ in (9.11) and simplifying yields

$$f'(x) = \frac{4b(x^2 - d)}{(x^2 + bx + d)^2}.$$

Setting $f'(x) = 0$ and keeping in mind that $x = \mathrm{Im}\, h > 0$ and $d > 0$ we obtain a critical number

$$x = \sqrt{d},$$

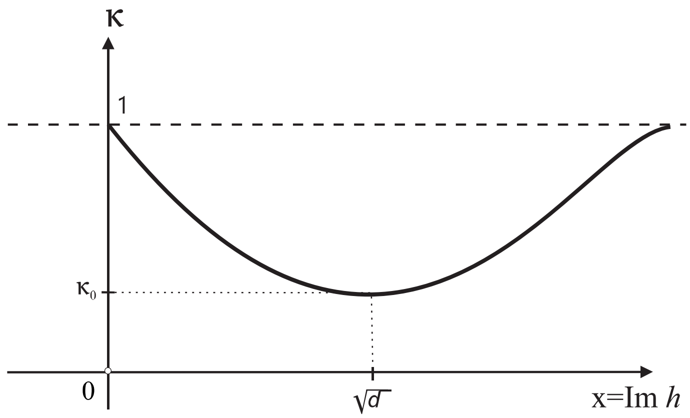


Fig. 9.1: Extremal $T_h$

that can be checked to be a point of minimum of $f(x)$ for all $x > 0$. Also, direct substitution gives

$$f(\sqrt{d}) = (\sqrt{d} - b)/(\sqrt{d} + b).$$

Consequently,

$$(\sqrt{d} - b)/(\sqrt{d} + b) \leq \kappa^2 < 1,$$

and hence

$$\sqrt{(\sqrt{d} - b)/(\sqrt{d} + b)} \leq \kappa < 1,$$

or, after backward substitution and simplification,

$$\sqrt{\frac{\sqrt{|m_\infty(i)|^2 - 2m_\infty(-0)\mathrm{Re}\; m_\infty(i) + m_\infty^2(-0)} + \mathrm{Im}\, m_\infty(i)}{\sqrt{|m_\infty(i)|^2 - 2m_\infty(-0)\mathrm{Re}\; m_\infty(i) + m_\infty^2(-0)} - \mathrm{Im}\, m_\infty(i)}} \leq \kappa < 1. \tag{9.12}$$

Thus, if $T_h$ is an extremal operator, then (9.12) holds, that is, the modulus $\kappa$ of von Neumann's parameter of $T_h$ is such that $\kappa_0 \leq \kappa < 1$, where $\kappa_0$ is given by (9.8). A sample graph of $\kappa$ as a function of $\mathrm{Im}\, h$ is shown in Figure 9.1. □

Now we will state and prove an inverse version of Theorem 9.1.

**Theorem 9.2.** *Let $\dot{A}$ be the same as in the statement of Theorem 9.1. If $\kappa_0 \leq \kappa < 1$, where $\kappa_0 \geq 0$ is given by (9.8), then there exists an extremal dissipative Schrödinger operator $T_h$, ($\dot{A} \subset T_h \subset \dot{A}^*$), of the form (9.5) with the modulus of its von Neumann's parameter equal to $\kappa$.*

Proof. Let $T_h$ be a Schrödinger operator of the form (9.5) with the same potential as $\dot{A}$ in (9.4). All we need is to show that there exists a value of $h$ that makes $T_h$ an extremal dissipative quasi-self-adjoint extension of $\dot{A}$ whose modulus of von Neumann's parameter is equal to the given $\kappa \in [\kappa_0, 1)$. Note that equation (9.10) holds for any $T_h$ with von Neumann's parameter $\kappa$. Setting

$$\mathrm{Re}\, h = -m_\infty(-0) = -m$$

in (9.10) will guarantee (see [4, 19, 20]) that $T_h$ is extremal and yield (9.11), that is

$$\kappa^2 = \frac{x^2 - 2bx + d}{x^2 + 2bx + d}, \tag{9.13}$$

where $x = \operatorname{Im} h$. Modifying (9.13) leads to the quadratic equation

$$x^2 - 2b\left(\frac{1+\kappa^2}{1-\kappa^2}\right)x + d = 0. \tag{9.14}$$

We are going to show that equation (9.14) has at least one real positive solution. For the sake of simplicity we set

$$\xi = (1+\kappa^2)/(1-\kappa^2)$$

and note that when $\kappa \in [\kappa_0, 1)$ we have $\xi \in [\xi_0, +\infty)$, where

$$\xi_0 = \left(1 + \frac{\sqrt{d}-b}{\sqrt{d}+b}\right) \Big/ \left(1 - \frac{\sqrt{d}-b}{\sqrt{d}+b}\right) = \frac{\sqrt{d}}{b}.$$

Consider the discriminant of the quadratic equation (9.14) as a function of $\xi$

$$D(\xi) = 4b^2\xi^2 - 4d.$$

Then its derivative $D'(\xi) = 8b^2\xi$ is always positive on $\xi \in [\xi_0, +\infty)$ indicating that $D(\xi)$ is an increasing function of $\xi \in [\xi_0, +\infty)$. Moreover, the direct check reveals that

$$D(\xi_0) = 4b^2\xi_0^2 - 4d = 0,$$

and hence $D(\xi)$ takes positive values on $\xi \in (\xi_0, +\infty)$. Applying the quadratic formula to equation (9.14) and taking into account that $b > 0$ and $\xi > 0$, we obtain that

$$x = \frac{2b\xi \pm \sqrt{4b^2\xi^2 - 4d}}{2} \tag{9.15}$$

yields at least one positive real solution which we can use. Therefore, the value of $h$ such that $\operatorname{Re} h = -m_\infty(-0)$ and $\operatorname{Im} h$ equal to the positive value of $x$ from (9.15) is the one that makes $T_h$ an extremal dissipative quasi-self-adjoint extension of $\dot{A}$. $\square$

The following example will provide an illustration to Theorems 9.1 and 9.2.

**Example.** Consider a minimal symmetric operator $\dot{A}$ of the form

$$\begin{cases} \dot{A}\, y = -y'' \\ y(1) = y'(1) = 0 \end{cases} \tag{9.16}$$

in the Hilbert space $\mathcal{H} = L^2[1, \infty)$. It is known [18] that $\dot{A}$ has deficiency indices $(1, 1)$ and is nonnegative. Consider also an operator

$$\begin{cases} T_h\, y = -y'' \\ y'(1) = hy(1). \end{cases} \tag{9.17}$$

It has been shown in [5] that in this case

$$m_\infty(z) = -i\sqrt{z} \quad \text{and} \quad m_\infty(i) = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i.$$

Consequently, $m_\infty(-0) = 0$ and $|m_\infty(i)| = 1$, and hence (9.8) yields

$$\kappa_0 = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}. \tag{9.18}$$

On the other hand, according to (9.7) we have the following formula representing the modulus of von Neumann's parameter of $T_h$ in (9.17) for any $h$

$$\kappa=\left|\frac{m_\infty(i)+h}{m_\infty(i)+\bar h}\right|=\left|\frac{\frac{1}{\sqrt2}-\frac{1}{\sqrt2}i+h}{\frac{1}{\sqrt2}-\frac{1}{\sqrt2}i+\bar h}\right|=\left|\frac{1-i+\sqrt2\operatorname{Re}h+\sqrt2\operatorname{Im}h\,i}{1-i+\sqrt2\operatorname{Re}h-\sqrt2\operatorname{Im}h\,i}\right|$$
$$=\sqrt{\frac{(1+\sqrt2\operatorname{Re}h)^2+(\sqrt2\operatorname{Im}h-1)^2}{(1+\sqrt2\operatorname{Re}h)^2+(\sqrt2\operatorname{Im}h+1)^2}}.$$

If we want our operator $T_h$ to be extremal, we set $\operatorname{Re}h=-m_\infty(-0)=0$ (see [4,19,20]) in the above formula. Then we get

$$\kappa_{ext}=\sqrt{\frac{1+(\sqrt2\operatorname{Im}h-1)^2}{1+(\sqrt2\operatorname{Im}h+1)^2}}=\sqrt{\frac{1+(\operatorname{Im}h)^2-\sqrt2\operatorname{Im}h}{1+(\operatorname{Im}h)^2+\sqrt2\operatorname{Im}h}},$$

where $\kappa_{ext}$ is the modulus of von Neumann's parameter of any extremal operator $T_h$ of the form (9.17). Also, according to Theorem 9.1 $\kappa_{ext}$ satisfies $\kappa_0\le\kappa_{ext}<1$, where $\kappa_0$ is given by (9.18). We can also directly describe the extremal operator $T_{h_0}$ whose von Neumann's parameter is $\kappa_0$ in (9.18). As we have shown this in the proof of Theorem 9.1, $\kappa_0$ corresponds to the critical value $\operatorname{Im}h=\sqrt d$, where $d$ is defined in (9.9). Thus,

$$h_0=-m_\infty(-0)+i\sqrt d=0+i\sqrt{(\operatorname{Re}m_\infty(i)-m_\infty(-0))^2+(\operatorname{Im}m_\infty(i))^2}=i|m_\infty(i)|=i.$$

Thus, the extremal accretive operator

$$\begin{cases} T_i\,y=-y'' \\ y'(1)=iy(1), \end{cases}\tag{9.19}$$

has modulus of its von Neumann's parameter equal to $\kappa_0$ of the form (9.18).

**Concluding remarks.** Formula (9.8) describing $\kappa_0$ plays an important role in the theory of conservative linear systems with Schrödinger operator. This topic is going to be discussed further in an upcoming paper (in preparation).

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**О ПАРАМЕТРЕ ФОН НЕЙМАНА ЭКСТРЕМАЛЬНОГО ОПЕРАТОРА ШРЁДИНГЕРА**

***С. В. Белый, Э. Р. Цекановский***

В настоящей работе мы изучаем связи между экстремальными акретивными квазисамосопряжёнными диссипативными расширениями неотрицательного симметрического оператора Шрёдингера с индексами дефекта $(1,1)$ в $L_2[\ell,+\infty)$ и их параметрами фон Неймана. Нами показано, что модуль соответствующего такому расширению параметра фон Неймана принадлежит интервалу $[\kappa_0,1)$, где для $\kappa_0\geq 0$ получена новая формула в терминах значений функции Вейля-Титчмарша $m_\infty(-0)$ и $m_\infty(i)$. Приведен пример иллюстрирующий полученные результаты.
***Ключевые слова:*** Оператор Шрёдингера, параметр фон Неймана, секториальнй оператор, экстремальный оператор, функция Вейля-Титчмарша.

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