Test 2

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1. Express each permutation as a product of disjoint cycles.

(a) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 2 & 4 & 1 \end{bmatrix}$,

(b) $(1,9,2,4)(1,7,6,5,9)(1,2,3,8)$.

2. Compute gfg^{-1} for each pair f, g.

(a) $f = (1,3,5,6)$; $\qquad g = (2,5,4,6)$,

(b) $f = (1,4)(2,3)$; $\qquad g = (1,2,3)$.

3. List all the elements of S_4, written as cycles, and find cyclic subgroups of S_4 that have three different orders.

4. List all the conjugate classes in S_4, find the c_a's, and verify the class equation.

5. If N is a normal subgroup of G and $a \in N$, show that every conjugate of a in G is also in N.

6. Prove that if $o(G) = p\,q$, $(p > q)$ where p and q are two distinct primes, then G is not simple (i.e. has a non-trivial normal subgroup).

7. Let $o(G) = p\,q$, $(p < q)$, p and q are two distinct primes. Show that if $p \nmid (q-1)$, then G is cyclic.

8. Show that in a group G of order p^2 any normal subgroup of order p must lie in the center of G.

9. Let G be a group of order 30. Show that a 3-Sylow subgroup or a 5-Sylow subgroup of G must be normal.

10. If G is a group of order 231, prove that the 11-Sylow subgroup is the center of G.